when $\theta = 0°$.

Given that $\theta = 0°$, $t = 0s$

TIME

$d\theta = 3t^2 dt$

$\int_0^\theta d\theta = \int_0^t 3t^2 dt$

$\theta = t^3$ rad

(1) $\theta = 45°$, $\pi/4$ rad

$\pi/4 = t^3$

$t = 0.9226 s$

$v = 0.766 m/s$, N, $\theta = 45°$

$\dot{t} = \ddot{t} = 0$

$\dot{\theta} = 3t^2 \Big|_{t=0.9226} = 2.554 rad/s$

$\ddot{\theta} = 6t \Big|_{t=0.9226} = 5.536 rad/s$

velocity

$V_r = \dot{r} = 0$

$V = \sqrt{V_r^2 + V_\theta^2}$

$= \sqrt{0^2 + 0.766^2} = 0.766 m/s$

Acceleration

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 0.3(2.554)^2 = -1.957 m/s^2$

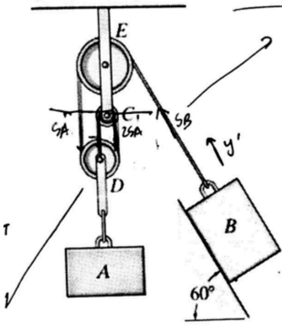
$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.3(5.536) + 0 = 1.66 m/s^2$

Kinematic:

Applying equation $s = s_0 + V_0 t + \frac{1}{2}a_c t^2$

$(\nwarrow +)$ $0.75 = 0 + 0 + \frac{1}{2} a_B (2^2)$

$a_B = 0.375 m/s^2$



Prob. 13–27

$2s_A + (s_A - s_B) = l$

$3s_A - s_B = l$

DERIVE

$3a_A - a_B = 0$

Eq (1)

$3a_A - 0.375 = 0$

$a_A = 0.125 m/s^2$

181 mA

Tension in pulleys is THE SAME throughout pulleys.

Block B

$\nwarrow + \sum F_{y'} = m a_{y'}$

$T - 5(9.81)\sin 60° = 5(0.375)$

$\boxed{T = 44.35 N}$

Block A

point B.

$T_A + V_A = T_B + V_B$

set up general eq...

$\frac{1}{2} m V_A^2 + m g h_A + \frac{1}{2} K x_A^2 =$

$\frac{1}{2} m V_B^2 + m g h_B + \frac{1}{2} K x_B^2$

A	B
$V_A = 0$	$V_B = ?$
$H_A = ?$	$H_B = 0$
$X_A = ?$	$X_B = ?$

$0 + [120 + 225] = \frac{1}{2}\left(\frac{20}{32.2}\right) V_B^2 + [0 + 2$

$V_B = 32.1 ft/s$

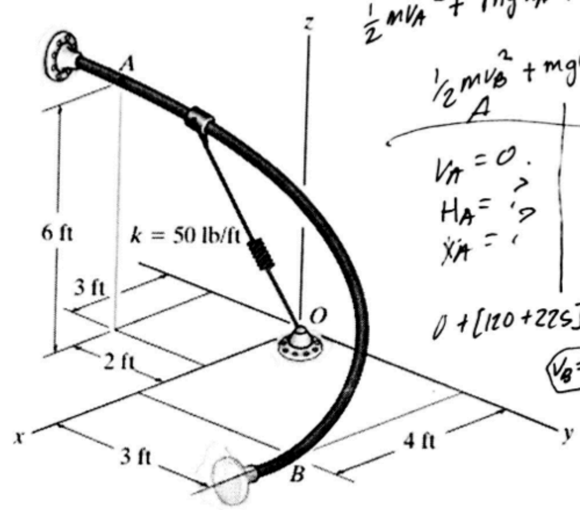


Fig. 16–7

i) CONSERVATION OF ENERGY:

Prob. 15–58

$T_1 + V_1 = T_2 + V_2$

$0 + 90 = \frac{1}{2}\left(\frac{15}{32.2}\right)(V_A)^2 + 0$

a) $(V_A)_1 = 19.66 ft/s = 19.7 ft/s$

ii) CONSERVATION of MOMENTUM

$m_A (V_A)_1 + m_B (V_B)_1 = m_A (V_A)_2 + m_B (V_B)_2$

$\left(\frac{15}{32.2}\right)(19.66) + 0 = \left(\frac{15}{32.2}\right)(V_A)_2 + \left(\frac{10}{32.2}\right)(V_B)_2$

Given $e = 0.3 = \frac{(V_B)_2 - (V_A)_2}{(V_A)_1 - (V_B)_1} = \frac{(V_B)_2 - (V_A)_2}{19.66 - 0}$

b) Solve: $(V_A)_2 = 9.435 ft/s$

$(V_B)_2 = 19.33 ft/s$

iii) PRINCIPLE OF WORK AND ENERGY

Ff

mg

$N_B = mg = 10 lb$

$F_f = \mu_k N_B = 4 lb$

$T_1 + \sum U_{12} = T_2$

$\frac{1}{2}\left(\frac{10}{32.2}\right)(19.33^2) + (-4 x_B) = 0$

c) $\boxed{x_B = 9.13 ft}$

Geometry:

$\frac{1}{2}x = r\cos\theta$

$\therefore x = 2r\cos\theta$

Calculus:

$x = 2r\cos\theta$

$\frac{dx}{dt} = -2r(\sin\theta)\frac{d\theta}{dt}$

$\boxed{v = -2r\omega\sin\theta}$

$\frac{dv}{dt} = -2r\frac{d\omega}{dt}\sin\theta - 2r\omega(\cos\theta)\frac{d\theta}{dt}$

$\boxed{a = -2r(\alpha\sin\theta + \omega^2\cos\theta)}$

$v\,dv = a\,ds$

$\int_0^v v\,dv = \int_0^s (8 - 2s)\,ds$

$\frac{v^2}{2}\Big|_0^v = (8s - s^2)\Big|_0^s$

$v = \sqrt{16s - 2s^2}$ m/s

$\boxed{V\Big|_{s=2} = \sqrt{16(2) - 2(2)^2} = \pm 4.90 m/s}$

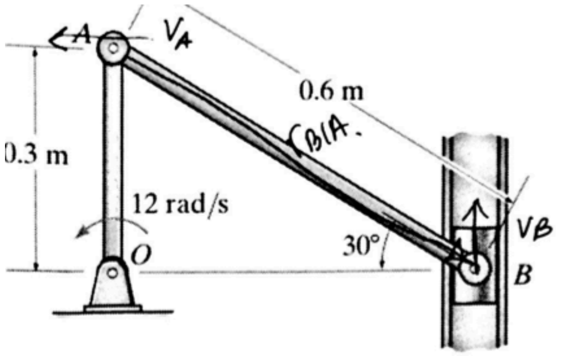
$= \frac{ds}{dv} \cdot a$

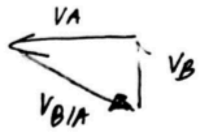
$\frac{dv}{ds} = 0$

$\frac{dv}{ds} = \frac{16 - 4s}{2\sqrt{16s - 2s^2}} = 0$

$16 - 4s = 0$

$\boxed{s = 4 m}$





F16–10

$$V_A = \omega_{OA} \times r_A$$
$$= (12 \text{ rad } \hat{k}) \times (0.3\text{m})\hat{j}$$
$$= \begin{vmatrix} i & j & k \\ 0 & 0 & 12 \\ 0 & 0.3 & 0 \end{vmatrix}$$
$$V_A = -3.6 \uparrow \text{ m/s}$$

$$V_B = V_A + \omega_{AB} \times r_{B/A}$$
$$V_B\hat{j} = (-3.6 \uparrow) + (\omega_{AB}\hat{k}) \times \langle 0.6\cos 30°, -0.6\sin 30° \rangle$$
$$V_B\hat{j} = [\omega_{AB}(0.6\sin 30°) - 3.6]\uparrow + \omega_{AB}(0.6\cos 30°)\hat{j}$$
$$\uparrow: 0 = \omega_{AB}(0.6\sin 30°) - 3.6$$
$$\hat{j}: V_B = \omega_{AB}(0.6\cos 30°)$$
$$\boxed{\omega_{AB} = 12 \text{ rad/s}} \quad \boxed{V_B = 6.24 \text{ m/s} \uparrow}$$

SOLUTION

$\omega = \omega_0 + \alpha_c t$

$\omega = 8 + 6(0.5) = 11$ rad/s

$v = r\omega; \qquad v_A = 2(11) = 22$ ft/s

$a_t = r\alpha; \qquad (a_A)_t = 2(6) = 12.0 \text{ ft/s}^2$

$a_n = \omega^2 r; \qquad (a_A)_n = (11)^2(2) = 242 \text{ ft/s}^2$

disk

$$\frac{ds}{dt} = \frac{5}{4 + s}$$

$$\int_5^s (4 + s)\, ds = \int_0^t 5\, dt$$

$$4s + 0.5s^2 - 32.5 = 5t$$

When $t = 6$ s,

$$s^2 + 8s - 125 = 0$$

Solving for the positive root

$$s = 7.87 \text{ m}$$

Also, the change in velocity is equal to the area under the a–t graph. Thus,

$$\Delta v = \int a dt$$

$$0 = \frac{1}{2}(3)(75) + \frac{1}{2}\left[\left(-\frac{1}{15}t' + 5\right)(t' - 75)\right]$$

$$0 = -\frac{1}{30}t'^2 + 5t' - 75$$

stopping car

Geometry: Here, $\frac{dy}{dx} = -0.00625x$ and $\frac{d^2y}{dx^2} = -0.00625$. The slope angle θ at point A is given by

$$\tan\theta = \left.\frac{dy}{dx}\right|_{x=80\text{ m}} = -0.00625(80) \qquad \theta = -26.57°$$

and the radius of curvature at point A is

$$\rho = \frac{[1 + (dy/dx)^2]^{3/2}}{|d^2y/dx^2|} = \left.\frac{[1 + (-0.00625x)^2]^{3/2}}{|-0.00625|}\right|_{x=80\text{ m}} = 223.61 \text{ m}$$

Equation of Motion: Applying Eq. 13–8 with $\theta = 26.57°$ and $\rho = 223.61$ m, we

$\Sigma F_t = ma_t; \qquad 800(9.81)\sin 26.57° - F_f = 800(3)$

$F_f = 1109.73 \text{ N} = 1.11 \text{ kN}$

$\Sigma F_n = ma_n; \qquad 800(9.81)\cos 26.57° - N = 800\left(\frac{9^2}{223.61}\right)$

$N = 6729.67 \text{ N} = 6.73 \text{ kN}$

$$3s_E + s_P = l$$

$$3v_E = -v_P$$

$(+\downarrow) \qquad v_P = v_E + v_{P/E}$

$$-3v_E = v_E + 6$$

$$v_E = -\frac{6}{4} = -1.5 \text{ m/s} = 1.5 \text{ m/s} \uparrow$$

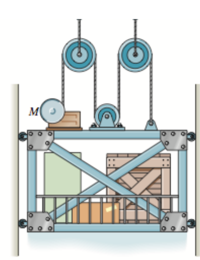
$(+\uparrow) \qquad v = v_0 + a_c t$

$$1.5 = 0 + a_E(2)$$

$$a_E = 0.75 \text{ m/s}^2 \uparrow$$

$+\uparrow \Sigma F_y = ma_y; \qquad 4T - 500(9.81) = 500(0.75)$

$T = 1320 \text{ N} = 1.32 \text{ kN}$



Kinematics: The speed of the block A and B can be related by using position coordinate equation.

$$s_A + (s_A - s_B) = l \qquad 2s_A - s_B = l$$

$$2\Delta s_A - \Delta s_B = 0 \qquad \Delta s_B = 2\Delta s_A = 2(3) = 6 \text{ ft}$$

$$2v_A - v_B = 0 \tag{1}$$

Equation of Motion: Applying Eq. 13–7, we have

$$+\Sigma F_{y'} = ma_{y'}; \qquad N - 60\left(\frac{4}{5}\right) = \frac{60}{32.2}(0) \qquad N = 48.0 \text{ lb}$$

Principle of Work and Energy: By considering the whole system, W_A which acts in the direction of the displacement does *positive* work. W_B and the friction force $F_f = \mu_k N = 0.2(48.0) = 9.60$ lb does *negative* work since they act in the opposite direction to that of displacement Here, W_A is being displaced vertically (downward) $\frac{3}{5}\Delta s_A$ and W_B is being displaced vertically (upward) Δs_B. Since blocks A and B are at rest initially, $T_1 = 0$. Applying Eq. 14–7, we have

$$T_1 + \sum U_{1-2} = T_2$$

$$0 + W_A\left(\frac{3}{5}\Delta s_A\right) - F_f\Delta s_A - W_B\Delta s_B = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B\ v_B^2$$

$$60\left[\frac{3}{5}(3)\right] - 9.60(3) - 10(6) = \frac{1}{2}\left(\frac{60}{32.2}\right)v_A^2 + \frac{1}{2}\left(\frac{10}{32.2}\right)v_B^2$$

$$1236.48 = 60v_A^2 + 10v_B^2 \tag{2}$$

Cam problem

Position Coordinates: From the geometry shown in Fig. *a*,

$$x_A = e\cos\theta + r$$

Time Derivatives: Taking the time derivative,

$$v_A = \dot{x}_A = -e\sin\theta\dot{\theta}$$

Since ω acts in the negative rotational sense of θ, then $\dot{\theta} = -\omega$. Thus, Eq. (2)

$$v_A = -e\sin\theta(-\omega) = e\omega\sin\theta \rightarrow$$

Taking the time derivative of Eq. (2) gives

$$a_A = \ddot{x}_A = -e\left(\sin\theta\ddot{\theta} + \cos\theta\dot{\theta}^2\right)$$

Since ω is constant, $\ddot{\theta} = \alpha = 0$. Then Eq. (3) gives

$$a_A = -e[\sin\theta(0) + \cos\theta(\omega^2)]$$
$$= -e\omega^2\cos\theta$$
$$= e\omega^2\cos\theta \leftarrow$$

The negative sign indicates that $\mathbf{a}_A$ acts towards the negative sense of x_A.

Moving circle with person

$$\mathbf{a}_D = \mathbf{a}_O + \dot{\Omega}\times\mathbf{r}_{D/O} + \Omega\times(\Omega\times\mathbf{r}_{D/O}) + 2\Omega\times(\mathbf{v}_{D/O})_{xyz} + (\mathbf{a}_{D/O})_{xyz} \tag{1}$$

Motion of moving reference	*Motion of D with respect to moving reference*
$\mathbf{a}_O = 0$	$\mathbf{r}_{D/O} = \{1\mathbf{i}\}$ m
$\Omega = \{0.5\mathbf{k}\}$ rad/s	$(\mathbf{v}_{D/O})_{xyz} = \{0.75\mathbf{j}\}$ m/s
$\dot{\Omega} = 0$	$(\mathbf{a}_{D/O})_{xyz} = \mathbf{0}$

Substitute the data into Eq.(1):

$$\mathbf{a}_B = 0 + (0)\times(1\mathbf{i}) + (0.5\mathbf{k})\times[(0.5\mathbf{k})\times(1\mathbf{i})] + 2(0.5\mathbf{k})\times(0.75\mathbf{j}) + \mathbf{0}$$
$$= \{-1\mathbf{i}\}\text{ m/s}^2 \qquad \textbf{Ans.}$$

(b)

$$\mathbf{a}_B = \mathbf{a}_O + \dot{\Omega}\times\mathbf{r}_{B/O} + \Omega\times(\Omega\times\mathbf{r}_{B/O}) + 2\Omega\times(\mathbf{v}_{B/O})_{xyz} + (\mathbf{a}_{B/O})_{xyz} \tag{2}$$

Motion of moving reference	*Motion of B with respect to moving reference*
$\mathbf{a}_o = \mathbf{0}$	$\mathbf{r}_{B/O} = \{3\mathbf{i}\}$ m
$\Omega = \{0.5\mathbf{k}\}$ rad/s	$(\mathbf{v}_{B/O})_{xyz} = \{0.75\mathbf{j}\}$ m/s
$\Omega = \mathbf{0}$	$(\mathbf{a}_{B/O})_{xyz} = -(a_{B/O})_n\mathbf{i} + (a_{B/O})_t\mathbf{j}$
	$= -(\frac{0.75^2}{3})\mathbf{i}$
	$= \{-0.1875\mathbf{i}\}$ m/s

Substitute the data into Eq.(2):

$$\mathbf{a}_B = \mathbf{0} + (\mathbf{0})\times(3\mathbf{i}) + (0.5\mathbf{k})\times[(0.5\mathbf{k})\times(3\mathbf{i})] + 2(0.5\mathbf{k})\times(0.75\mathbf{j}) + (-0.1875\mathbf{i})$$
$$= \{-1.69\mathbf{i}\}\text{ m/s}^2 \qquad \textbf{Ans.}$$

Equations of Motion: Since the rod rotates about a fixed axis passing through point A, $(a_G)_t = \alpha r_G = \alpha(0.3)$ and $(a_G)_n = \omega^2 r_G = (5^2)(0.3) = 7.5\text{ m/s}^2$. The mass moment of inertia of the rod about its mass center is $I_G = \frac{1}{12}mL^2 = \frac{1}{12}(12)(0.6^2) = 0.36\text{ kg}\cdot\text{m}^2$. Writing the moment equation of motion about point A and referring to Fig. *a*,

$$+\Sigma M_A = \Sigma(M_k)_A; \quad -12(9.81)(0.3) = -0.36\alpha - 12[\alpha(0.3)](0.3)$$

$$\alpha = 24.525\text{ rad/s}^2 = 24.5\text{ rad/s}^2 \qquad \textbf{Ans.}$$

This result can also be obtained by applying $\Sigma M_A = I_A\alpha$, where $I_A = \frac{1}{3}ml^2 = \frac{1}{3}(12)(0.6^2) = 1.44\text{ kg}\cdot\text{m}^2$. Thus,

$$+\Sigma M_A = I_A\alpha; \qquad -12(9.81)(0.3) = -1.44\alpha$$

$$\alpha = 24.525\text{ rad/s}^2 = 24.5\text{ rad/s}^2 \qquad \textbf{Ans.}$$